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MATHEMATICAL MODEL OF ERROR OF ANALOG-DIGITAL CONVERSION OF BIOOBJECT RESPONSE AT EXCITATION

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Abstract. Improvement of mathematical model of biological objects response to the low intensity excitation is the problem. Solution of this problem will ensure improvement of biotechnological synthesis and preclinical screening of biological objects

Keywords: biological object; low intensity excitation; mathematical model; analog-digital conversion; estimation.

Introduction. For controlling and diagnosing of functional state, for correction functions of biological object the method of electrophysiological active research of this object often use. In particularly the use of low intensity excitation for stimulation of biological object (information influence on biological object) is effective enough. The tendencies to reduce of light intensity is caused by necessity as comfortable conditions for biological object so and (in concordance with Weber-Fechner's law) increasing informativeness of response of that biological object [1, 2].

Objective. Main requirements for rate parameter of response estimation and reliability of the assessments to ensure high quality biomedical research (its effectiveness) is accompanied by the need to consider ADC errors probability distribution function values which may depend on time, that is, the error is not stationary, random, with property of the stochasticity its frequency characteristics.

Materials and Methods. Responses on the excitation of the biological objects adequately displayed as well as continuous functions. After selection, the responses subjected to analog-digital conversion (ADC) for further processing as a sequence of binary codes. Series of excitations executed for response estimation and ensemble of responses of biological objects are obtained as result.

Discussion. Synchronization of the responses (strings of codes) in the ensemble to ensure the coherence of these responses is main stage for automated estimation of the response and should be carried out with minimal complexity. For automated ensuring of the coherence of responses is necessary should determine the beginning of the active parts - oscillating process, and executed cyclic shift of each response in ensemble so as to response began with the beginning of the active phase (for ensuring the reliability of response estimation) . The main difficulty of this procedure are in optimum estimation of the latent part ending moment of the response (for further statistical analysis (ensemble averaging).

In mathematical modeling used the fact that biological objects: a) is a dynamic object; b) contains linearity; c) behaves stochastically; d) adequately modeled by versions of stochastic differential equations in partial derivatives. In practice, essential results receive by simplification on the correct bases of such assumptions, (for example, use of the usual differential equations, received from a bioobject biosignals). For active physiologic researches are using biosignals as response (reaction) of the bioobject on a test excitation. Last interpreted as core of linear, time-invariant differential operator [3, 4], degree no less than the second. The response is simulated as oscillation, with consolation function that belongs to biophysical (own - time-domain), i.e, space of trajectories [5, 6]. Such functions belongs to the linear space of continuous functions in a specified range with the norm L^2 [7].

Optimization of the parameters of analog-digital conversion. Process of the ADC of the responses is the selection of samples with discretization frequency. This process requires time-consuming (ADC aperture time) during which the response changes. To reduce the complexity the code-impulse ADC are used without devices sample and hold on input (one of realization its contains in converters, such as the type ADuC 8**). For these ADCs is essential ensure the discretization frequency and the number of ADC bits. This must be adequate to errors of spectrum and quantization of investigated responses.

In the technical literature may be apply the ADC with conditions [8]:

$$\left|\frac{dx(t)}{dt}\right|_{\max} \leq \frac{c}{T_{\mathrm{d}}}, \qquad (1)$$

where $x(t), t \in [0, \Theta]$ – response on ADC input; $[0, \Theta]$ – duration of the response length; T_{d} – period of discretization; $c = \frac{|x_{\min} - x_{\max}|}{2^n}$, c – quantization step; n – quantity of the binary position at the presentation of the point by the binary complementary code.

Criterion (1) is valid for deterministic, differentiable responses $x(t)$ and for a limited class of differentiable random processes (quasi determinate).

Stronger ratio that describes eligibility condition for ADC applications is inequality:

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$$f(\theta, x) \leq c, \qquad (2)$$

where $f(\theta, x) = \max\limits_{\substack{t_1, t_2 \in [t, t+\theta] \\ t \in [0, \Theta]}} |x(t_1) - x(t_2)|$ – module of the function continuity $f(t)$, θ – conversion time. Obviously, $\dfrac{f(\theta, x)}{\theta} \leq \max\limits_{t \in [0, \Theta]} \left| \dfrac{dx(t)}{dt} \right|$.

For random processes, particularly for Gaussian when the density distribution is not finite, and also for not differentiable random processes, application of the criterion (1) requires to prove. Using representation (1) and (2) makes it possible to provide choice of the parameters T_d, n of ADC, taking into account the correlation and spectral properties of responses in the ensemble, and provide statistical representativeness of samples.

To determine the period of discretization T_d is necessary to know about the maximum rate of increase or spectrum width of response. This are related. In particular, for the practical applications there is a formula:

$$\Delta\tau = 0{,}35 / \Delta f, \qquad (3)$$

where Δf – spectrum width; $\Delta\tau$ – time of the response increase in the largest dynamic range of it values.

For estimation of the response spectrum width, according to the its stochasticity, the estimation of width of spectral power density $S(\omega)$ used. For this estimated autocorrelation function and its Fourier transform (Winer Hinchen's theorem):

$$R_{xx}(\tau) = \int_{-\Theta}^{+\Theta} x(t) x(t+\tau) dt$$

$$S^2(\omega) = \int_0^{\tau} R_{xx}(\tau) e^{-j\omega\tau} d\tau.$$

For computer simulation of the response is necessary have the form for represent of effect of response digitization and coding of the received discrete points during the ADC. For this aim may be use (2, 3).

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Computer modeling of an error of analog-digital conversion. Earlier noises in channels of digital processing of signals were analyzed [8]. For modeling of response discretization can be that discretization presented by expression

$$x_k \triangleq x(kT_d) = \int_{\Theta} x(t)\delta(t - kT_d)dt,$$

where $\delta(\cdot)$ — delta function, $k = \overline{1, K}$, $\Theta = KT_d$.

The value of the discrete noise modeled by expression

$$\xi_k = x_k^1 - x_k^2,$$

where $x_k^1 = \Xi_1\{\exp(-\alpha kT_d)\sin(2\pi kT_d / \mu)\}$, $x_k^2 = \Xi_2\{\exp(-\alpha kT_d)\sin(2\pi kT_d / \mu)\}$, $\Xi_{1,2}[\cdot]$ — quantization operators with different weight c of least significant bit with n-bits binary code of number with fixed comma; in case T_d — period of discretization, then $\mu = \dfrac{T_d}{T_s}$, $\mu' = \dfrac{T_d + \theta}{T_s}$, $0 < \theta < T_d$, $k = \overline{0, \infty}$.

Operators of quantization Ξ of points of responses are designated then so:

$$\Xi_{\bullet} : x(kT_d) \in \mathbb{R} \to \hat{x}(kT_d) \in \{e_m, m = \overline{0, n-1}\}_k, e_m \in GF(2^n).$$

Inverse operator $\Xi'_{\bullet} : \hat{x} \in GF(2^n) \to x \in \mathbb{R}$ does not exist therefore error of mathematical model:

$$\xi_k = x(kT_d) - \hat{x}(kT_d).$$

Digital resolution of ADC $c = \dfrac{A}{2^n}$, where $e_0, e_{1\cdot i} \in GF(2^n)$, zero and primitive elements of finite and extended Galois field with properties $e_{1i} \wedge e_{1j} = \begin{cases} e_0, i \neq j; \\ e_{1i}, i = j; \end{cases}$ and $e_0 \wedge e_{1i} = e_0$, $i = \overline{1, n}$. In case $\alpha_i \in \{e_0, e_{1i},\ i = \overline{1, n}\}$ determined by the recursive algorithm whole part of number

$$y_i = x(mT_d),\ y_{i+1} = Ent\left(\frac{y_i}{2}\right);$$

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$$\alpha_{i+1} = \begin{cases} e_0, & (y_{i+1} < 1) \vee (y_i - 2y_{i+1} = 0) \\ e_{1i}, & (y_{i+1} = 1) \vee (y_i - 2y_{i+1} = 1) \end{cases},$$

where $Ent(\cdot)$ — whole part of number, $i = \overline{0,\ n-1}$. Then

$$\Xi_1[x(\cdot)] = \begin{cases} \bigcup_i^n e_{1i}, & |x(\cdot)| \geq A; \\ \bigcup_i^n (\alpha_i \wedge e_{1i}), & \frac{c}{2} \leq |x(\cdot)| < A; \\ e_0, & |x(\cdot)| < \frac{c}{2}. \end{cases}$$

We will designate that $\beta_{im} = \begin{cases} 0, \alpha_{im} = e_0; \\ 1, \alpha_{im} = e_{1i}. \end{cases}$. Then reverse conversion

$$\Xi_1' :\ \hat{x}_m = c\sum_{i=1}^{n} \beta_{im} 2^{i-1}.$$

Computer modeling of the response error depends on the rate of change it:

$$|\xi_\bullet| \in \begin{cases} (0, c), & \left.\frac{dx(t)}{dt}\right|_{\max} \leq c / T_d \\ (0, q \cdot c), & \left.\frac{dx(t)}{dt}\right|_{\max} > c / T_d \end{cases},$$

where $q \in \mathbb{Z}$ — whole numbers system.

Depending on the type of number μ (natural number, rational, irrational) the cycle value will vary $\nu = [\phi(Q(\mu))]^{-1}$, where Q — is the supplement to the whole part of number, $\phi(\cdot)$ — Riemann function. Therefore, the value ξ_k, are taken through a period $N = Ent(\mu^{-1})$, where $Ent(\cdot)$ — whole part of number, may be grouped — $\xi_{i\eta}$, $\eta = \overline{1, N}$, $i = \overline{1, \nu}$. Because T_s i T_d incommensurable periods, appears the "crowding" of phase of points response. For a finite interval of monitoring T range $\xi(iT_d)$ depending on the class of number μ breaks on condition of isolation $Q(\frac{m}{T_d}) = \frac{\mu}{\nu}$, (where $m = T_d Fr(\mu)$, $Q(\cdot) = 1 - Fr(\cdot)$, $Fr(\cdot)$ — fractional part of number) into subsets power M_Ω. It depended from the value T. For one cycle M_Ω takes values:

$$M_\Omega = \begin{Bmatrix} \mu \\ \nu Ent(\mu) \\ \infty \end{Bmatrix}, \ \frac{T_s}{T_d} \in \begin{Bmatrix} \mathrm{N} \\ \mathrm{R} \\ \mathrm{Q} \end{Bmatrix},$$

where Ω — power of subset M.

These values are representative and making the succession of vectors. This is a implementation of point error $\xi(m,\omega)$, $m = \overline{1,M}$, $M = Ent(\mu)$, $\omega \in \Omega$ — item of subset element Ω. This was the basis for the designation of indicator display of indexes I: $\{i\} \xrightarrow{I^i_{m,\omega}} \{m,\omega\}$. In resulting received the sets of realizations (ensemble)

$$\xi(iT_d) \xrightarrow{I^i_{m,\omega}} \xi^{(n)}_{p_1,\dots,p_n}\{m,\omega\},$$

where $p \in K$. Put that the permutation of components of each *m*-vector does not change it properties.

Results. The received expressions give the chance to model errors of ADC, function of distribution of probabilities of values by means computer modelling at depend on time, that is, the error is non-stationary and random, with property of repeatability of characteristics of stochasticity. For example, of its correlation function if the discretization frequency and quantity of bits - are non-optimal. Otherwise, the error is stationary white noise.

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