# THE HEAT TRANSFER MODEL FOR DEVICE OF CARDIOPULMONARY BYPASS WITH THE IMPLEMENTATION OF THE MECHANISM OF FREE CONVECTION HEAT IN THE MYOCARDIUM

**Valentyna A. Danilova [1], Vladislav V. Shlykov [1]**

[1] Department of Biomedical Engineering National Technical University of Ukraine, Kiev, Ukraine

**Abstract.** The heat transfer models for device of cardiopulmonary bypass in the form of a system of ordinary differential equations are presented. The numerical heat transfer model in the myocardium, which takes into account the initial temperature distribution and free convection mechanism in the myocardium. The numerical model allows us to estimate the temperature of the process parameters of hypothermia and hyperthermia heart and to investigate changes in the gradient of the temperature on the surface of the myocardium at the time of registration of thermal images of the heart.

**Keywords**: thermogram; myocardium; temperature distribution; vascular pathology.

**Introduction.** The heart is a complex of pump-muscular system, the functions of which depend on the contractile properties of the myocardium material. The myocardium has a large functional reserve (especially the left ventricle), or the ability to maintain a stable pumping function and a high ability to adapt to stress [1]. The heat transfer during extracorporeal cardiopulmonary bypass (CPB) is due to the heat exchange between the blood and the water in the heat exchanger device of cardiopulmonary bypass (DCB) and to the heat exchange between the blood and the body of the patient's in the circulatory system. In accordance with the protocol of CPB blood is first using a special catheter enters the oxygenator to the pump (centrifuge pump) which replaces lung function, and then from the oxygenator blood moves to the heat exchanger, which lowers the blood temperature and across a catheter (silicone tube) directed in the patient's system circulation [1]..

**Objective.** The heat balance model that developed for extracorporeal cardiopulmonary bypass allowed the evaluate the dynamics of the cooling process and warming the heart and determine the temperature gradient at the surface of the myocardium during the registration process thermograms hypo- and hyperthermia and obtain the distribution of temperature profiles, which enables diagnosis of ischemic lesions in the myocardium

**Materials and methods.**

**The heat balance equation for cardiopulmonary bypass**

For the CPB the blood flow is maintained at $2,2 - 2,4 \, L/(min \cdot m^2)$ and temperature at $28 - 35 \, °C$ during moderate hypothermia, and $16 - 26 \, °C$ during deep hypothermia. Reduction of the patient's body temperature leads to reduced oxygen demand for bodies. To perform the heat balance the heat exchange in the body must be equal to its heat transfer. At rest, the magnitude of the human body heat is about $75 \, kcal/g$ at about $36,6 \, °C$. When heating blood the amount of heat that is absorbed or excreted of blood, calculated using the formula:

$$Q_{lic} = c_{liq} m_{liq} (T_{lic} - T_{out}), J,$$

where $c_{liq}$ – the specific heat capacity of the blood, $c_{liq} \approx 4200 \, J/(kg \cdot s)$, $(T_{lic} - T_{out})$ – the temperature difference at heating blood, $°C$, $m_{liq}$ – the mass of cooled blood, $kg$.

The heat balance equation involving DCB for cooling and heating of blood will look:

$$Q_{lic} + Q_{heart} = Q_{dbc},$$

where $Q_{heart}$ – the amount of heat emitted or absorbed by the myocardium (heart) for cooling or heating of blood, J.

$Q_{dbc}$ – the amount of heat that is released or absorbed in the DCB, J.

Obviously, the combustion of the blood the temperature difference is $(T_{lic} - T_{out}) > 0$ and correspondingly the amount of heat is $Q_{lic} > 0$, and when the cooling blood the temperature difference is $(T_{lic} - T_{out}) < 0$ and $Q_{lic} < 0$ – blood releases energy in the DCB.

The terms in equation of the heat balance describe the contribution to the energy balance of the environment: the blood, the heart, the body, the air. The amount of heat which is transferred from the inner layer to the outer layer of the myocardium is described of the Fourier law [2]:

$$Q_{heart} = -K \frac{T_2 - T_1}{l} S_{heart} \cdot t,$$

$$T_1 = T_{heart} = \frac{T_1' + T_2'}{2}$$ –the temperature of the inner wall of the heart,

$T_1' = T_0$ – the initial value of the blood temperature in the DCB,

$T_2' = T_{liq}$ – the final cooling temperature of blood in the DCB,

$K$ – the thermal conductivity of the myocardium,

$S_{heart}$ – the area of the myocardium,

$l$ – the thickness of the myocardium.

The amount of heat that must be expended to cool the blood in the DCB from $T_1'$ to $T_2'$ can be found from the expression:

$$Q_{dcb} = (m_{lic} \cdot c_{lic} + m_{dcb} \cdot c_{dcb}) \cdot (T_1' - T_2'),$$

$m_{lic}$, $m_{dcb}$ – the mass of blood and the mass of material in the DCB, respectively, $kg$,

$c_{lic}$ – the specific heat capacity of the blood, $J/(kg \cdot K)$,

$c_{bca}$ – the specific heat capacity of the material in the DCB, $J/(kg \cdot K)$.

Thus, it is possible to obtain a mathematical model in the form of a conventional differential equation of the 1st-order that describes the heat exchange between the blood and the heart:

$$\begin{cases} \dfrac{dQ_{heart}}{dt} = \dfrac{1}{R_q}(T_{heart} - T_{out}) \\ \dfrac{dT_{heart}}{dt} = -r(T_0 - T_{lic}) \end{cases},$$

where $T_0 = T_{heart}\,|_{t=0}, K$ at time $t = 0$.

$R_q = \dfrac{1}{\alpha}$ – the thermodynamic resistance of the myocardium, $\dfrac{K}{W}$,

$r = \dfrac{1}{(m_{lie}c_{lie} + m_{dcb}c_{dcb}) \cdot R_q}$ – the coefficient cooling of the myocardium,

$T_{out} = T_{heart}\,|_{t_{i-1}}, K$ – the temperature in the preceding step $(i-1)$ the heat exchange for time $t = t_{i-1}$.

Given the temperature blood in the DCB and tissue myocardium, the temperature body of the patient and the air in the operating room, the model of heat exchange can be represented as the system of ordinary differential equations: the system (1) describes the heat exchange between he fluids blood-DCB, the system (2) describes the heat exchange between the fluids blood-heart, system (3) describes the heat exchange between the fluids air-body and the fluids body-heart.

Thus, it is the model of heat exchange is described systems of the differential equations:

$$\begin{cases} \dfrac{dQ_{heart}}{dt} = \dfrac{1}{R_q}(T_{heart} - T_{out1}), \\ \dfrac{dT_{heart}}{dt} = \dfrac{1}{(m_{lic}c_{lic} + m_{bca}c_{bca})}\left(\dfrac{dQ_{lic}}{dt} - \dfrac{dQ_{heart}}{dt}\right) - \dfrac{dT_{lic}}{dt} \end{cases},\quad (1)$$

$$\begin{cases} \dfrac{dQ_{lic}}{dt} = \dfrac{1}{R_q}\left(T_{lic} - T_{out2}\right), \\[2mm] \dfrac{dT_{lic}}{dt} = \dfrac{1}{\left(m_{lic}c_{lic} + m_h c_h\right)}\left(\dfrac{dQ_{heart}}{dt} - \dfrac{dQ_{lic}}{dt}\right), \end{cases} (2)$$

$$\begin{cases} \dfrac{dQ_{body}}{dt} = \dfrac{1}{R_q}\left(T_{body} - T_{out3}\right), \\[2mm] \dfrac{dT_{body}}{dt} = \dfrac{1}{\left(m_{lic}c_{lic} + m_b c_b\right)}\left(\dfrac{dQ_{heart}}{dt} - \dfrac{dQ_{body}}{dt}\right), \\[2mm] \dfrac{dQ_{air}}{dt} = \dfrac{1}{R_q}\left(T_{air} - T_{out4}\right), \\[2mm] \dfrac{dT_{air}}{dt} = \dfrac{1}{\left(m_{air}c_{air} + m_b c_b\right)}\left(\dfrac{dQ_{body}}{dt} - \dfrac{dQ_{air}}{dt}\right) \end{cases} (3)$$

The temperatures $T_{out1}, T_{out2}, T_{out3}, T_{out4}$ correspond to the values temperature at the previous step $(i-1)$ for the heat exchange between the blood and the objects to interact (DCB material, myocardium, body, air).

### The model of heat exchange with the mechanism of free convection heart the myocardium

The mass of the myocardium can be calculated by the formula:

$$m_{heart} = \rho_{heart} \cdot V_D = \rho_{heart}(V_{ext} - V_{int}),$$

where $\rho_{heart} = 1{,}05 - 1{,}2\, g/cm^3$ – the average value of the density of the myocardium,

$V_D = V_{ext} - V_{int}$ – end-diastolic volume of the myocardium (the left ventricle), equal to the difference of external and internal volumes.

The normal end-diastolic volume (EDV) is about 160 ml and the myocardial mass of about 105 g, but against the background of the development of coronary heart disease EDV significantly decreases or increases within 150-198 ml, and respectively, and changes of the myocardial mass of 105-168 g.

It is also known that the amount of heat which is transmitted through the monolayer of the myocardium wall defined by the formula [2]:

$$Q_{heart} = \lambda_{heart} \dfrac{T_{int} - T_{ext}}{l} S_{heart} \cdot t = \alpha_{heart}(T_{int} - T_{ext}) \cdot S_{heart} \cdot t,$$

where $\lambda_{heart} = 0{,}7\dfrac{W}{m \cdot K}$ – coefficient of the myocardial thermal conductivity,

$\alpha = \dfrac{\lambda_{heart}}{l}$ –heat transfer coefficient, $\dfrac{W}{m^2 \cdot K}$,

$T_{\text{int}}, T_{ext}$ – the temperature of the inner and outer surfaces of the myocardium, $K$ ,

$l$ – the thickness of the myocardial wall, $m$ ,

$S_{heart}$ – the wall surface area, $m^2$ ,

$t$ – the time of the heat exchange, s.

In turn, the amount of heat that must be expended to cool the heart can be found from the expression:

$$Q'_{heart} = m_{heart} \cdot c_{heart} \cdot (T_1 - T_2) ,$$

where $c_{heart} = 3{,}2 \cdot 10^3 \dfrac{J}{kg \cdot K}$ – the specific heat capacity of the myocardium,

$T_1, T_2$ – the initial and final temperature of the wall of the myocardium for hypothermia of the heart, $K$ .

A substantial increase of the wall thickness of the myocardium leads to an increase in the thermal resistance to zone of the atherosclerotic damage wall of the heart muscle. The equation of the heat balance for during the heat exchange between the inner and outer walls of the myocardium has the form:

$$Q_{heart} = Q'_{heart} \text{ or } \alpha_{heart} \cdot (T_{\text{int}} - T_{ext}) \cdot \Delta t = m_{heart} \cdot c_{heart} (T_1 - T_2) ,$$

where $\Delta t$ – the duration of the process hyperthermia of the heart, $s$ .

From this equation the coefficient cooling of the myocardium in a unit interval of time $\Delta t$ is equal to:

$$r = \frac{\alpha_{heart}}{m_{heart} \cdot c_{heart}} = \frac{T_1 - T_2}{T_{\text{int}} - T_{ext}} .$$

Accordingly, the change in thermal resistance of the myocardium can be expressed in terms of the temperature of the inner wall ( int ), which are cooled by the blood flowing from the heart-lung machine. And the outer wall temperature ( $ext$ ), are controlled by the thermal imager:

$$R_q = \frac{1}{m_{heart} \cdot c_{heart}} (\frac{T_{\text{int}} - T_{ext}}{T_1 - T_2}), \frac{K \cdot m^2}{W} .$$

Or relative to end-diastolic volume of the myocardium:

$$R_q = \frac{1}{\rho_{heart} \cdot c_{heart} \cdot V_D} (\frac{T_{\text{int}} - T_{ext}}{T_1 - T_2}),$$

where $T_1, T_2$ – the initial and final temperatures of hypothermia of the heart, which must be achieved by use the machine of an artificial heart-lung.

The thermal resistance $R_q$ and the coefficient cooling $r$ of the myocardium can be estimated based on a model of heat exchange in the local myocardial site which is implemented in the modelling system MSC Patran and MSC Sinda 2012.

The free convection coefficient between the three-dimensional objects - the myocardium and coronary vessels corresponds to the natural model of the laminar flow (ID = 701 for Convection Correlation Lib) across the surface with the thickness $d$ and the characteristic length $L$ can be found from the expression:

$$h = \frac{\lambda_{heart-liq}}{L} \cdot N_u ,$$

where $N_u^{\frac{1}{2}} = 0,825 + \dfrac{0,387 R_a^{\frac{1}{6}}}{\left[ 1 + \left( \dfrac{0,492}{P_r} \right)^{\frac{9}{16}} \right]^{\frac{5}{27}}}$ – the Nusselt number,

$R_a = G_r \cdot P_r$ – the Rayleigh number ($0,1 < R_a < 10^{12}$),

$P_r = \dfrac{\mu_{liq} \cdot c_{liq}}{\lambda_{heart-liq}}$ –the Prandtl number,

$G_r = \dfrac{L^3 \rho_{liq} g \beta \Delta T}{\mu_{liq}}$ –the Grashof number,

$\lambda_{heart-liq} \approx 2000 \dfrac{W}{m \cdot K}$ – the thermal conductivity at the interface of the blood-myocardium,

$\mu_{liq} \approx 1,89 \cdot 10^{-5} \dfrac{N \cdot s}{m^2}$ – the dynamic viscosity of blood,

$c_{liq} \approx 3,65 - 3,77 \cdot 10^3 \dfrac{J}{kg \cdot K}$ –the specific heat capacity of the blood,

$\rho_{liq} \approx 1,0 - 1,05 \cdot 10^3 \dfrac{kg}{m^3}$ –the blood density,

$\beta_{liq} \approx 0,0035 - 0,0039 \dfrac{1}{K}$ – the coefficient of thermal expansion of blood,

$\Delta T = (T_1 - T_2)$ – the temperature difference between the outer and inner surface of the body, $K$ ,

$l$ – the length of the body (myocardium), $m$ .

In this model, the characteristic size $L$ of heat transfer is assumed to be equal the length of the infarction site:

$$L = \frac{V_D}{S_{heart}} \cong l \text{ , } m \text{ ,}$$

where $V_D = dl^2$ –diastolic volume, $m^3$ ,

$S_{heart} = l^2$ –the area of the critical section of the piece of myocardium, $m^2$ .

When using a model of heat transfer for the problem of cooling of the myocardium, the temperature of the internal surface – coronary T1=5 $^\circ C$ , and the temperature of the outer surface - not cooled infarction T2=35 $^\circ C$ ,that by solving the problem of stationary convection of the laminar flow across the border blood-myocardium surface gives respectively the value of convection coefficient $h_1 \approx 4{,}70\dfrac{W}{m^2 \cdot K}$ и

$h_2 \approx 4{,}79\dfrac{W}{m^2 \cdot K}$ to establishing the heat balance.

### The model of the heat transfer for cardiopulmonary bypass

According to the above model of the heat transfer for extracorporeal cardiopulmonary bypass, the program model in Matlab has the form shown in Figure 1.
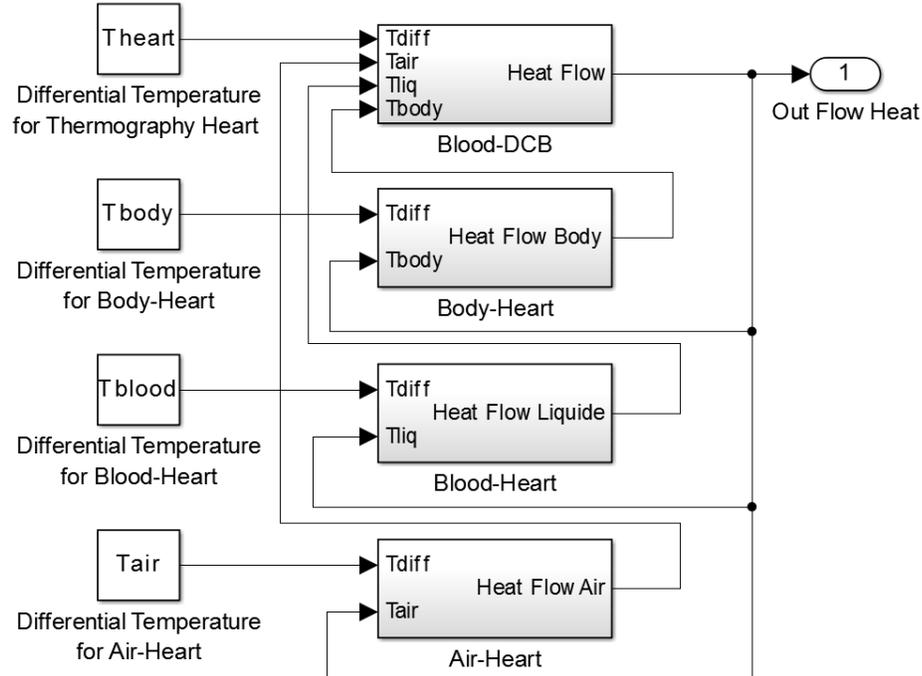


Fig. 1: Block diagram of the mathematical model of the heat transfer for the process of cooling blood in Matlab.

The program model in Matlab for each system of the differential equations $\dfrac{dQ}{dt} = \dfrac{1}{R_q}(T_1 - T_2)$ and

$\dfrac{dT}{dt} = \dfrac{1}{(m_1 c_1 + m_2 c_2)}\left(\dfrac{dQ_1}{dt} - \dfrac{dQ_2}{dt}\right)$ at using Simulink library elements has the form shown in Figure 2.
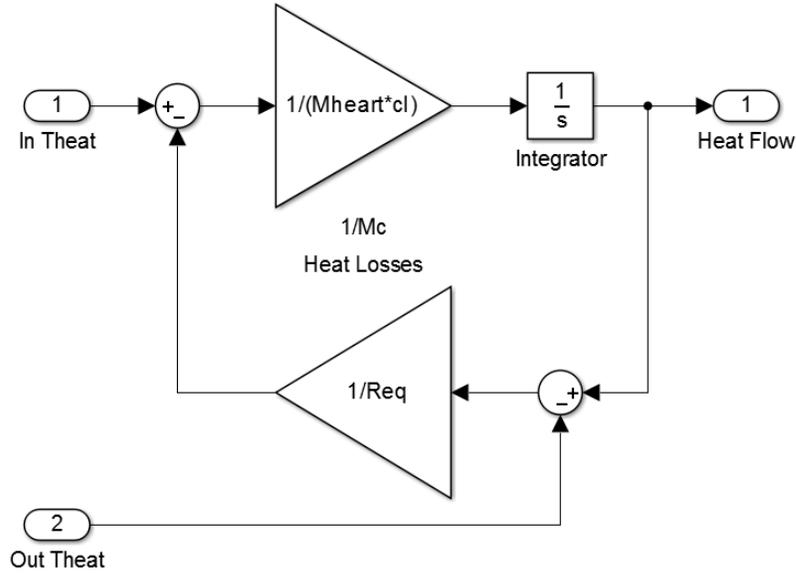


Fig.2: Block diagram of the mathematical model of the heat transfer for each system of the differential equations in Matlab.

The function of approximate lowering blood temperature by heat exchange in the DCB has the form shown in Figure 3.
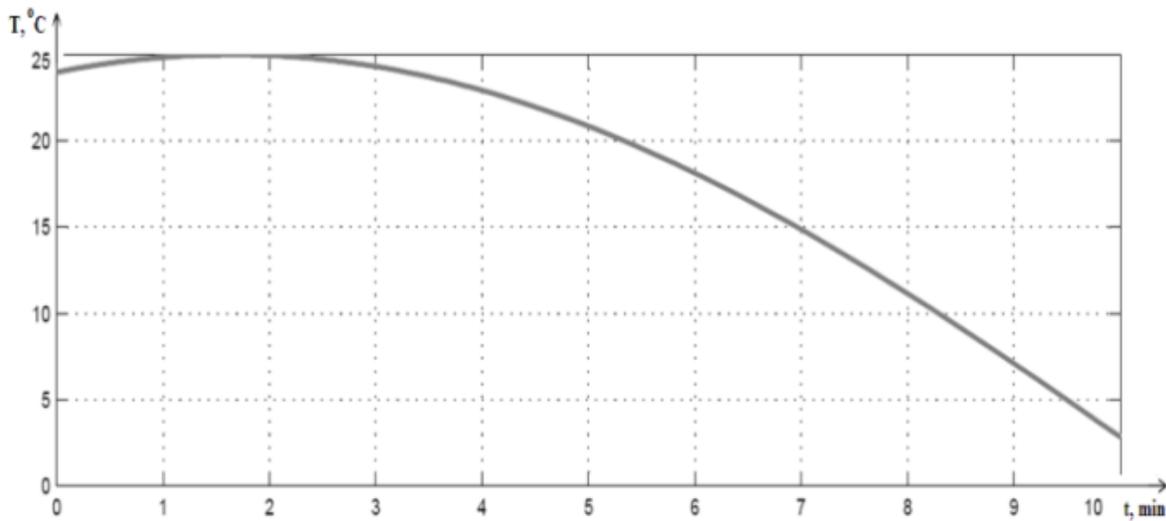


Fig.3: The process of step lowering blood temperature by heat exchange

The programming model of heat transfer that calculated using DCB function allows assessing the depth of cooling and rewarming of the heart at any given time during extracorporeal cardiopulmonary bypass [3].

The use of additional cooling for the hearts - the cooling of the infarction with ice on surface, which is at a temperature $T_{liq} \approx 1°C$, that to reduce uneven distribution of temperature of the heart for it is cooled with cardiopulmonary bypass.An example of a numerical model of heat transfer, which is employs the heat convection between the cube of ice and the surface of the myocardium is shown in Figure 4 (a, b).
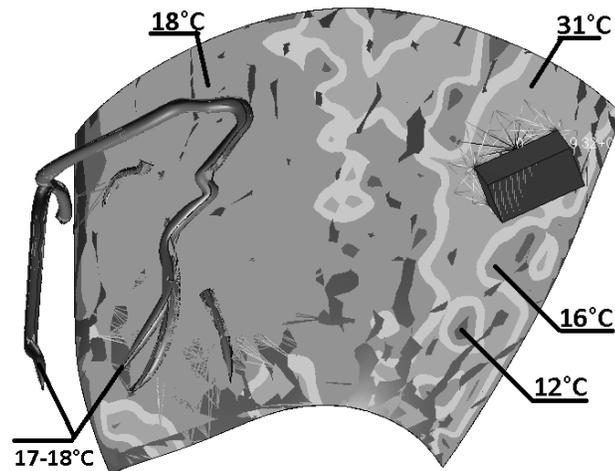


Fig. 4 a): The distribution of temperatures on the surface of the myocardium, which is further cooled with ice at a temperature $T_{liq} \approx 1°C$
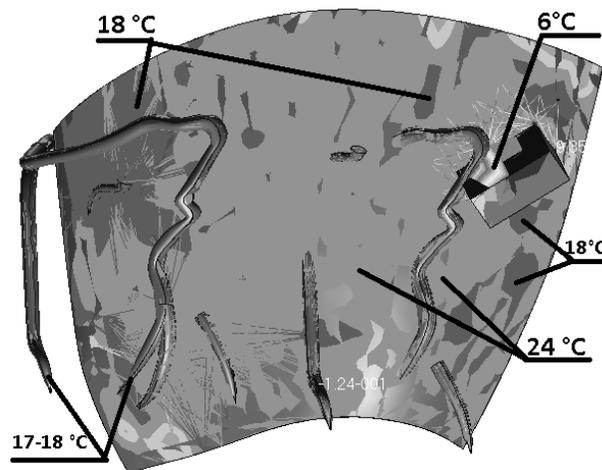


Fig. 4 b): The distribution of temperatures on the surface of the myocardium, which is further cooled with ice at a temperature $T_{liq} \approx 1°C$.

### The results of applying the model

The examples of infrared images of the heart with the most pronounced temperature gradients in the right and left ventricles, which corresponds to the beginning of the process of hypothermia, the full cooling of the myocardium and the ending of the process hyperthermia are presented, respectively, in the Figure 5 (a, b).
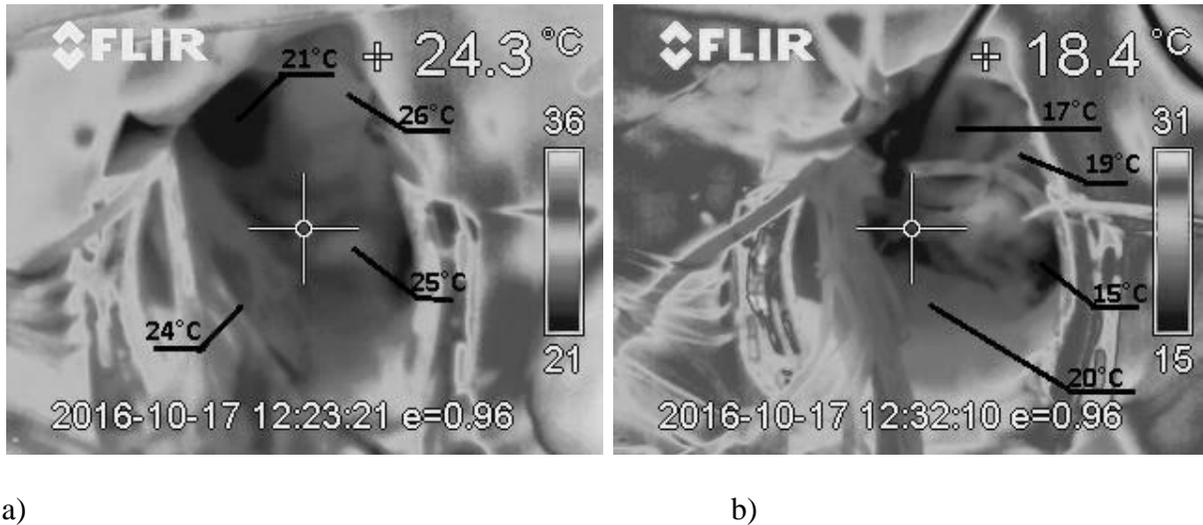


a)                                                                                              b)

Fig. 5: Thermograms of the heart for condition with cardiopulmonary bypass: a) To cooled myocardial for temperature T= 24 – 26 $^\circ C$ , b) To cooled myocardial for temperature T= 18 – 19 $^\circ C$ .

At the examples of the heart thermograms presented a maximum gradient of temperatures on the surface of the myocardium at the beginning of the process of hypothermia $\Delta T_A \approx 2,0°C$ , and at the ending for cooled myocardium $\Delta T_B \approx 1°C$ .

Reducing the temperature gradient between the areas with uneven cooling of the myocardium to the value $\Delta T_B \approx 1°C$ is explained by the establishment of the heat balance for chilled heart.And well explained by switching off the heart from the circulation for application of cardiopulmonary bypass. For a given the heart thermograms the rate of cooling of the myocardium of calculated for hypothermia process is equal:

$$r_A = \frac{T_2 - T_1}{T_{\text{int}} - T_{ext}} = \frac{26 - 24}{36 - 32} = 0,5 \text{ ,}$$

where $T_{\text{int}}, T_{ext}$ – the temperature of the upper and lower (outer) surface of the heart.

The calculated cooling rate $r$ values are in the range from 0,3 to 0,6, that indicating no significant temperature inhomogeneities of the temperature gradient at the surface of the myocardium.

For experimental verification of heat transfer model with a thermal imager Flir i7 at intervals of 1 minute were recorded thermographic images of the myocardium of the heart in the process of cooling. The initial and final stage of the cooling process of the heart in the form of infrared images of the myocardium and the surgical field displayed in Figure 6 (a, b):
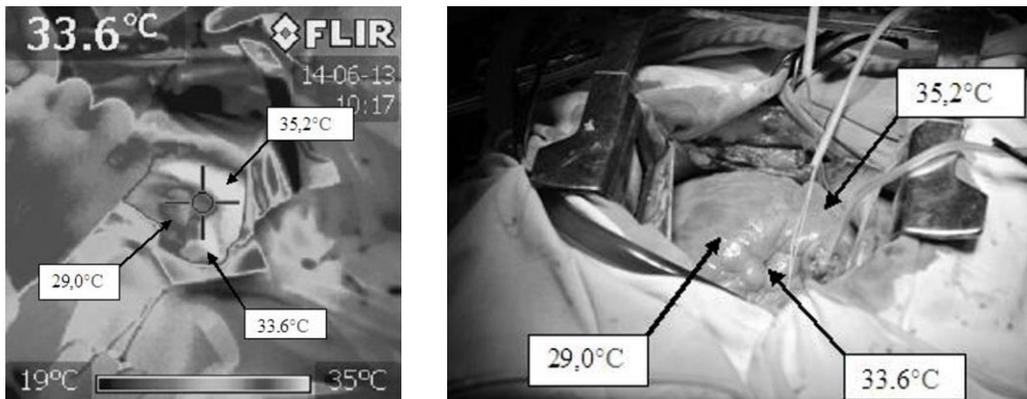


Fig. 6 a): Heart with temperature of cooling myocardium from $33,6\,°C$ to $10,3\,°C$



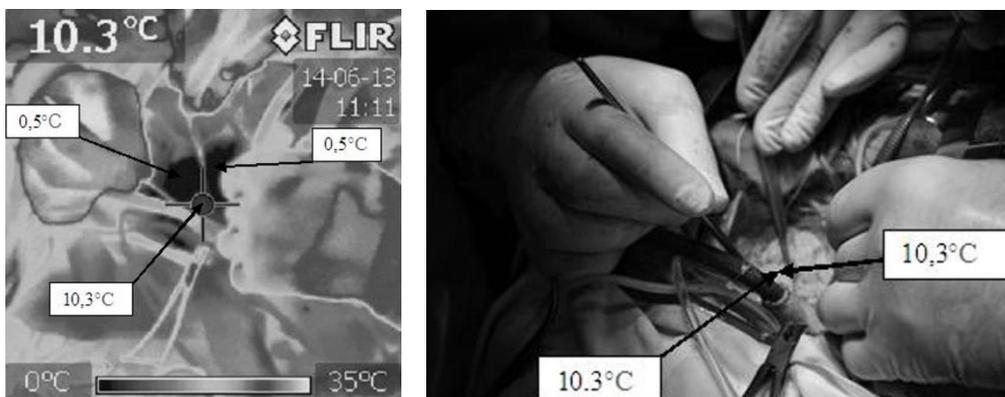Fig. 6 b): Heart with temperature of cooling myocardium from $33,6\,°C$ to $10,3\,°C$

At the beginning of hypothermia process the temperature of area for the heated myocardium was $33,6\,°C$ and in the final stage was lowered to $10.3\,°C$. The indications DCB at the end of the process hypothermia are determined that the temperature of open-heart surgery was maintained at $17\,°C$. The temperature distribution on the surface of the myocardium during cooling the cardiac at the readings DCB equal $17\,°C$ is shown in Figure 7.
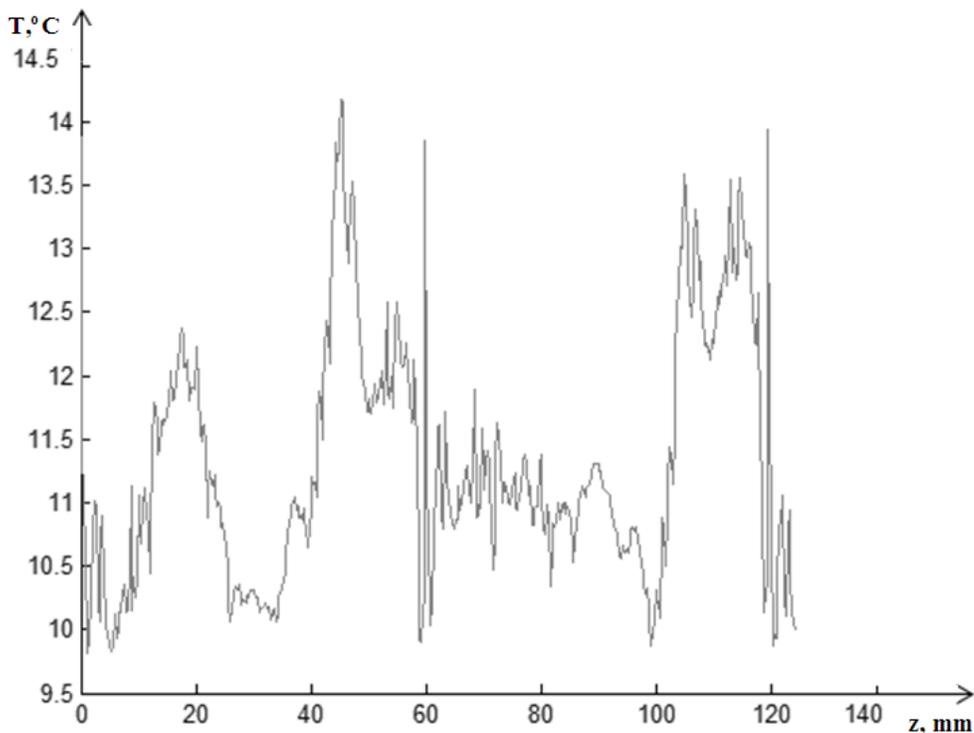
Fig.7: The temperature distribution on the surface of the myocardium upon cooling to $14 - 10\,^{\circ}C$

The temperature profile is constructed for chilled the heart and indicates the minimum temperature of myocardium $9,8 - 10,3\,^{\circ}C$ and the greatest difference between the maximum and minimum temperatures of around $3 - 4\,^{\circ}C$ .Therefore, temperature gradient on the surface of the myocardium before and after cooling of the heart is stable indicator which probably can be used as a diagnostic criterion in determining ischemic areas on the surface of the myocardium.

**Conclusions.** Thus, the heat balance model that developed for extracorporeal cardiopulmonary bypass allowed the evaluate the dynamics of the cooling process and warming the heart and determine the temperature gradient at the surface of the myocardium during the registration process thermograms hypo- and hyperthermia and obtain the distribution of temperature profiles, which enables diagnosis of ischemic lesions in the myocardium. The model implemented for heat exchange processes for hypothermia and hyperthermia that allows the temperature gradient to evaluate on the surface of the myocardium and calculate the coefficient cooling of the infarction. In other, the numerical heat transfer model in the myocardium allows to determine a possible the presence of ischemic myocardial lesions on the surface.

**Conflict of interest statement:** The authors state that there are no conflicts of interest regarding the publication of this article.

**ORCID**

V. V. Shlykov, OrcID 0000-0001-8836-4658, http://orcid.org/0000-0001-8836-4658,

V. A. Danilova, OrcID: 0000-0003-3009-6421, e-mail valnaa@ukr.net

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